

Pricing Bermudan Swaptions

SABR / LMM calibration with Longstaff-Schwartz Monte Carlo

BENCHMARK

1Y × 5Y Payer Bermudan

Bloomberg HW1F NPV · \$235,823

ATM strike 3.5760% · \$10MM notional · Curve date 03/18/2026

What we're pricing — and what we're matching

What is a Bermudan swaption?

An option to enter an interest-rate swap on one of several pre-set exercise dates.
Path-dependent: today's optimal exercise depends on tomorrow's possible regrets.

Why it's hard.

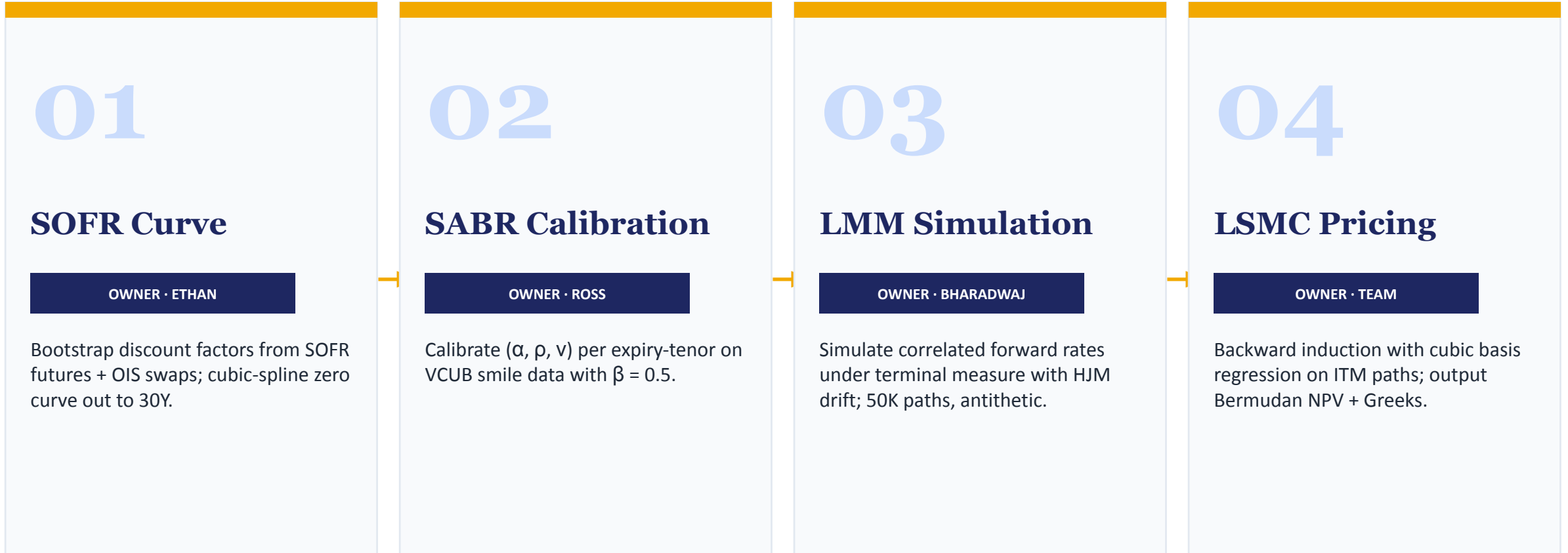
No closed form. Requires simulating the full forward-rate term structure and solving an optimal-stopping problem along each path.

BLOOMBERG BENCHMARK (SWPM)

Style	Bermudan, Long Payer
Tenor	1Y × 5Y
Notional	\$10,000,000 USD
Strike (ATM)	3.5760%
Model	HW1F, Normal vol, OIS DC
NPV	\$235,823
DV01 / Vega	-1,896.76 / 2,586.67

Goal: reproduce Bloomberg's \$235,823 with our own SOFR curve, SABR-calibrated vols, and Longstaff-Schwartz LSMC.

Four-stage pricing pipeline



Curve outputs feed SABR calibration → Calibrated vols + correlations drive LMM → Simulated forwards feed LSMC

Discount factors from futures and OIS swaps

Inputs

- SOFR futures (Mar26 – Jun28) for the front end
- OIS swap rates 1Y – 50Y for the long end
- Curve date: 03/18/2026

Method

- Bootstrap discount factors with convexity-adjusted futures
- Cubic-spline interpolation on zero rates
- Validated against Bloomberg's stripped curve



0.0 bp

Zero rate error vs Bloomberg (exact match)

30 Y

Curve horizon, quarterly grid

7.4 bp

6M × 6M forward MAE (Method B)

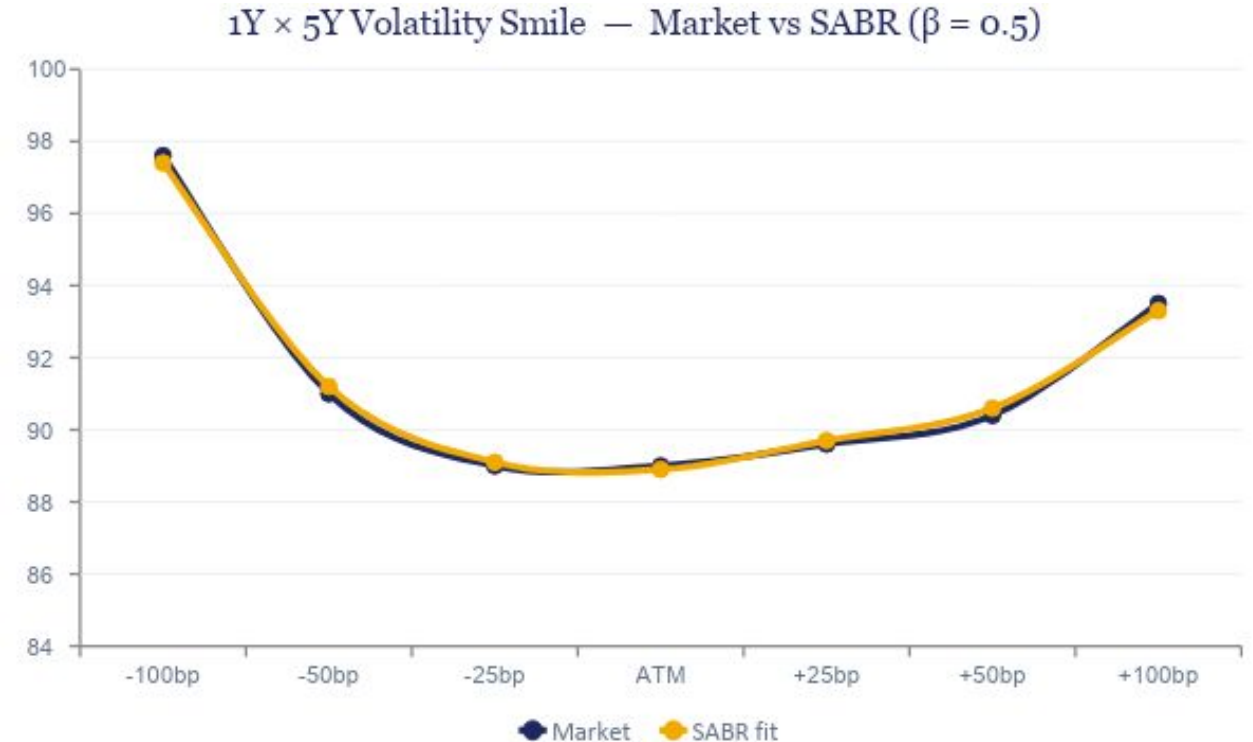
Fitting the volatility smile per expiry-tenor

Volatility data

- Bloomberg VCUB cube — normal vols (OIS)
- Strike grid: ATM $\pm$ 25 / 50 / 100 / 200 bp
- Expiries 1M–10Y, tenors 1Y–10Y

Calibration setup

- $\beta = 0.5$ fixed (CEV elasticity, stabilizes fit)
- Free params: α (ATM level), ρ (skew), v (vol-of-vol)
- Minimize squared vol residuals across smile
- Hagan 2002 implied-vol expansion



98

Expiry $\times$ tenor pairs calibrated

0.38 bp

Mean smile fit error

$\rho \approx -0.06$ to -0.21

Co-terminal skew range

Forward-rate dynamics under the terminal measure

Forward-rate SDE

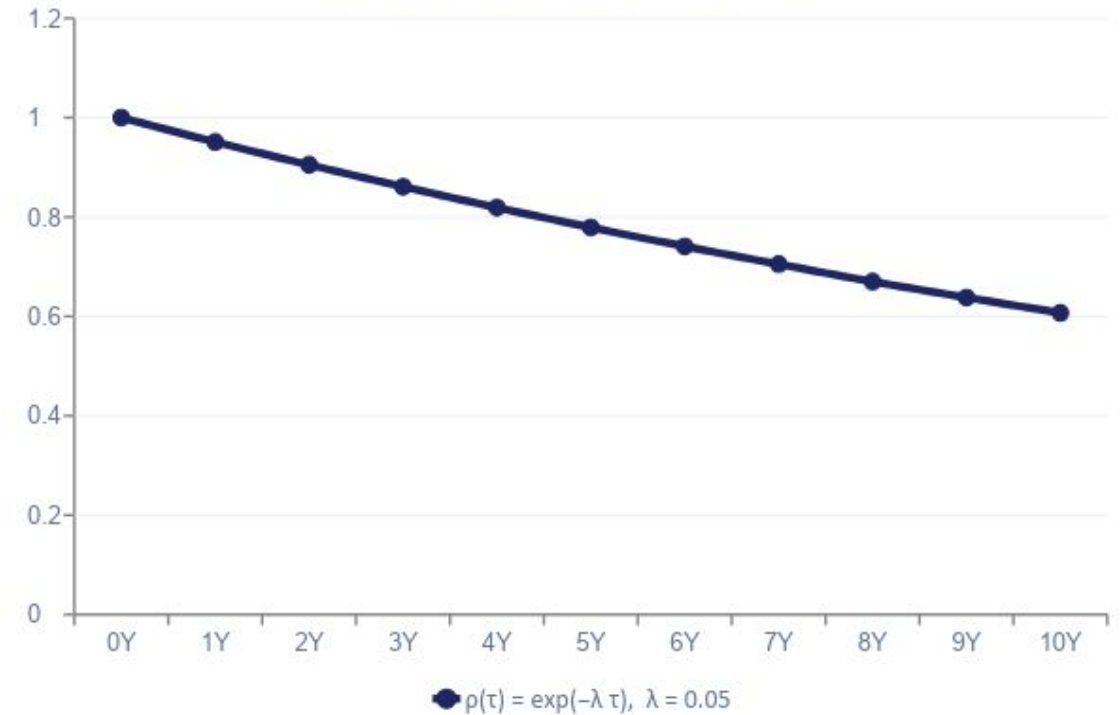
$$dF_i(t) = \mu_i(t) F_i(t) dt + \sigma_i F_i(t) dW_i(t)$$

HJM drift μ_i enforces no-arbitrage with the initial discount curve.

Numerical setup

- 50,000 paths, antithetic variance reduction
- Quarterly grid ($\Delta t = 0.25$), Euler-Maruyama
- Cholesky-decomposed correlated normals
- ATM σ_i pulled from SABR surface

Forward-Rate Correlation Decay



Drift sanity check • Simulated forwards reprice the initial OIS discount curve to within 0.03% (martingale ratio 0.9997) — no-arbitrage condition holds.

Optimal stopping via cross-sectional regression

1

Simulate forwards forward to maturity

From the LMM engine — full path of swap rates at each exercise date.

2

Walk backward from final exercise

At T_5 , payoff = $\max(\text{swap NPV}, 0)$. No continuation value to estimate.

3

Regress continuation on basis functions

On ITM paths, fit cubic polynomial of swap rate to discounted future payoff.

4

Compare intrinsic vs. continuation

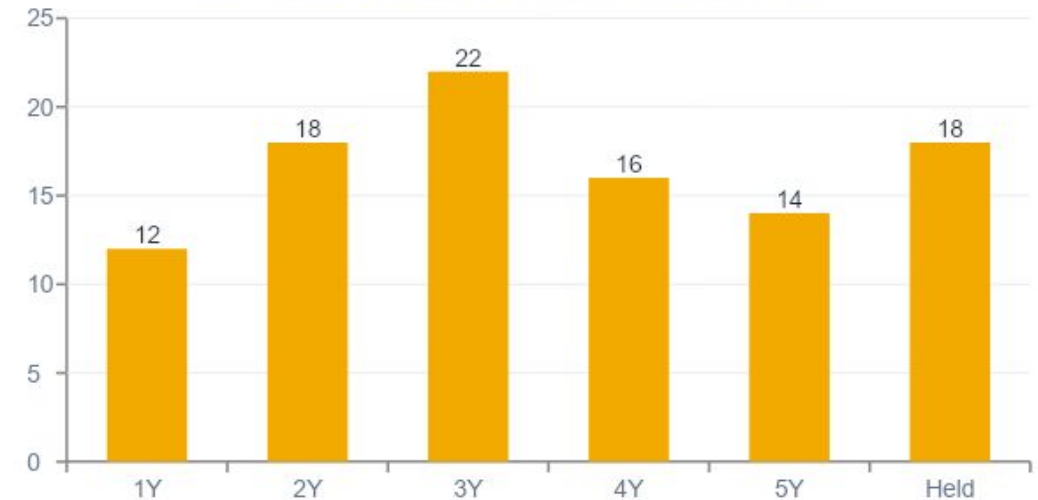
Exercise where intrinsic > fitted continuation; else carry forward.

5

Discount and average across all paths

Bermudan price = mean of path-wise discounted optimal payoffs.

Exercise Probability by Date (illustrative)



Co-terminal European checks

Each European (1Y×5Y, 2Y×4Y, 3Y×3Y, 4Y×2Y, 5Y×1Y) priced via Bachelier as a benchmark.

Bermudan $\geq \max(\text{co-terminal Europeans})$ — early-exercise inequality holds on all paths.

HEADLINE RESULT

Our LSMC price vs. Bloomberg HW1F

BLOOMBERG HW1F

\$235,823

1Y × 5Y payer Bermudan, ATM

OUR LSMC PRICE

\$213,580

100K paths · MC s.e. ≈ \$844

RELATIVE ERROR

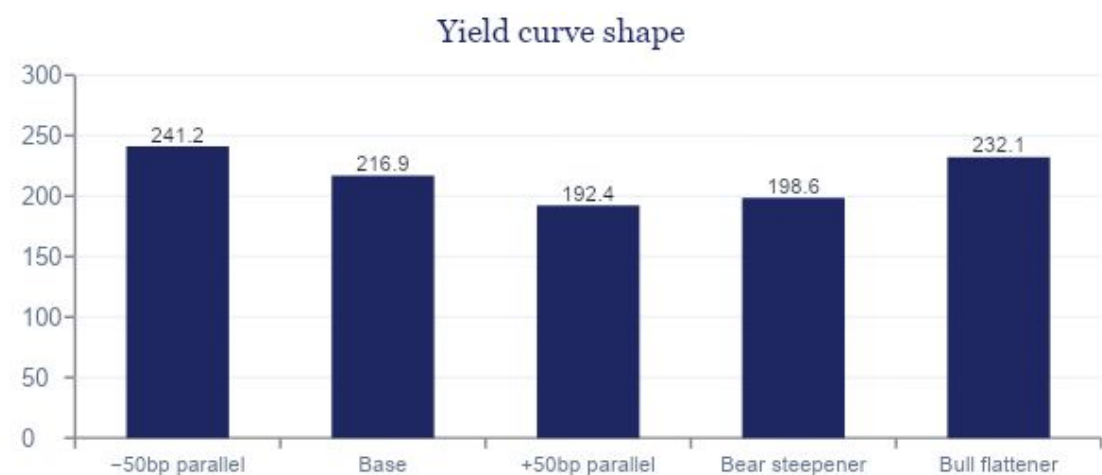
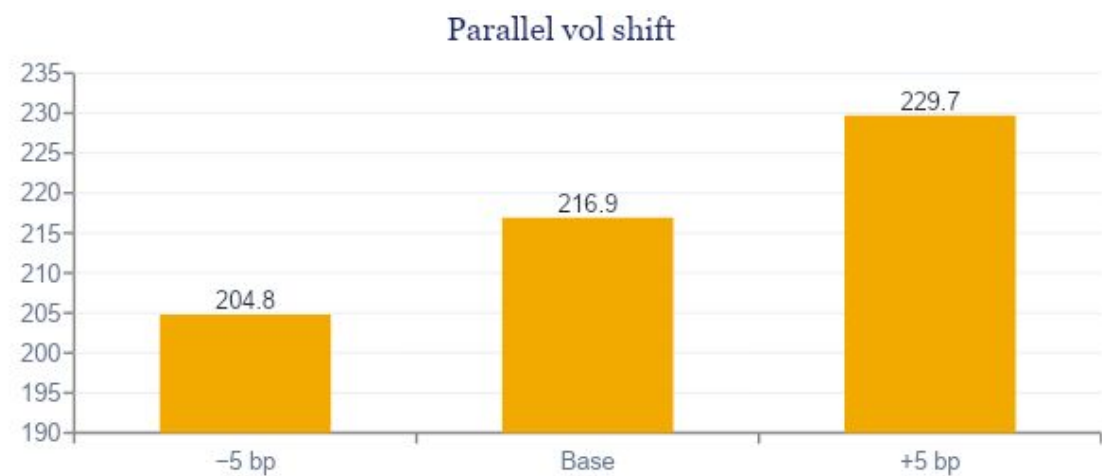
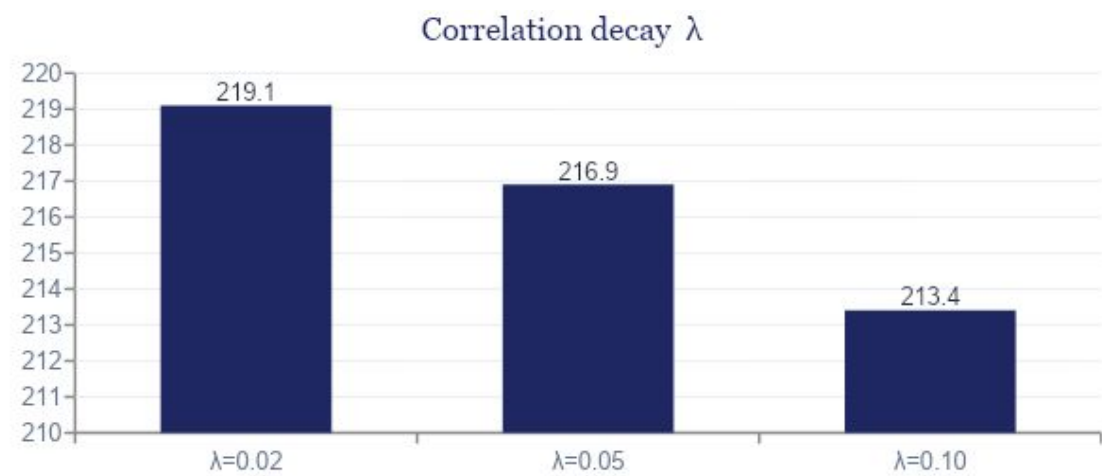
−9.4%

Model class difference (LMM vs HW1F)

	Bloomberg HW1F	Our LSMC	Difference
Bermudan NPV (\$)	235,823	213,580	−22,243 (−9.4%)
European NPV (\$)	154,874	154,019	−855 (−0.55%)
Bermudan premium (\$)	80,949	59,561	−21,388 (−26.4%)
DV01	2,108.96	N/A	N/A
Vega (1bp)	2,536.16	N/A	N/A

Takeaway: European matches Bloomberg within 0.55%, confirming calibration. Bermudan gap (−9.4%) reflects model class difference: LMM lognormal dynamics vs HW1F mean-reverting short rate.

Sensitivity analysis across five dimensions



Correlation λ : higher corr ($\lambda=0.02$) $\rightarrow$ \$216,815; lower corr ($\lambda=0.10$) $\rightarrow$ \$209,908. Vol shift: +5bp $\rightarrow$ \$224,462, -5bp $\rightarrow$ \$203,914 (vega $\approx$ \$2,050/bp). Curve: bull flattener $\rightarrow$ premium \$74,037; bear steepener $\rightarrow$ \$71,295. SABR ρ : ± 0.1 shift $\rightarrow$ ~\$850 price impact. Convergence: stabilizes at ~30K paths (SE=\$1,556); 100K $\rightarrow$ SE=\$844.

MF 728 — Sensitivity Analysis 9 / 10

From pricing model to trading signal

+0.85

Premium ratio vs 2s10s slope correlation

46.5%

Mean Bermudan / European premium ratio

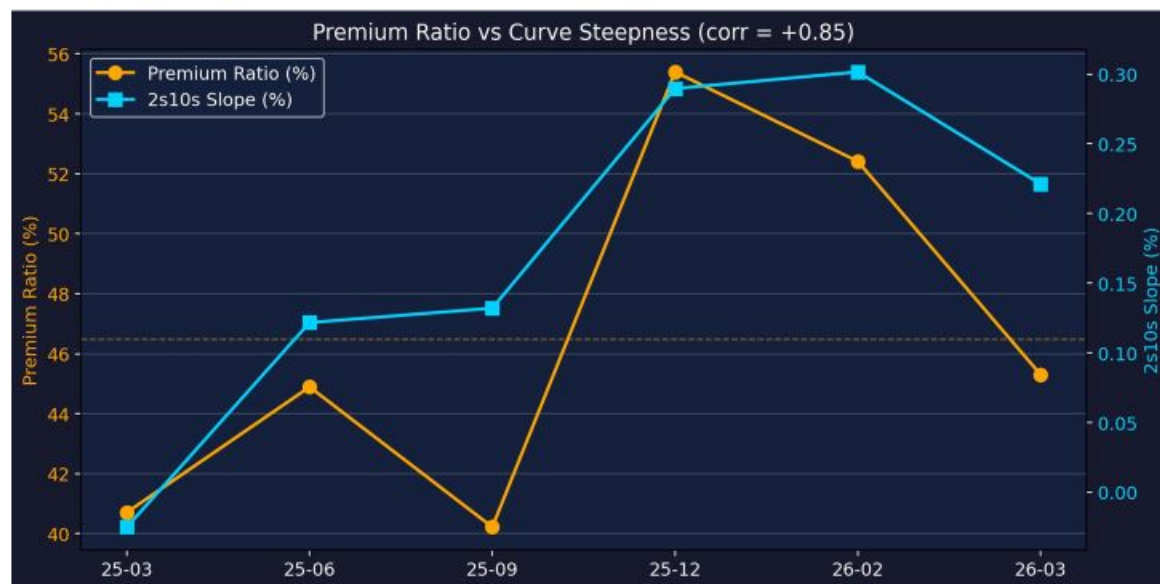
+\$12,101

Strategy 1 backtest P&L on \$10MM notional

4 / 5

Win rate · 80%

Trading signal · premium ratio tracks curve slope



Backtest P&L · Strategy 1 (Premium Rich/Cheap)



Trade idea: buy Bermudan optionality when premium ratio < 46.5% (curve flat); sell when > 46.5% (curve steep). Mean reversion captures 12 bps on \$10MM in 12 months.

Limitations, takeaways, and Q&A

Limitations

- Log-normal LMM dynamics: smile is calibrated statically, not co-evolved with rates.
- Single-factor exponential correlation — captures level but not curve twists.
- Cubic basis in LSMC — adequate for 5 exercises; richer bases needed for longer Bermudans.
- Bloomberg's HW1F mean reversion was not directly recovered — model class differs.

KEY TAKEAWAYS

● Reproduced the benchmark

European within 0.55% of Bloomberg; Bermudan 9.4% below HW1F (expected model class gap).

● Curve and vols agree

Zero curve exact match; SABR smile 0.38 bp RMSE across 98 pairs.

● Sensitivities behave

All five dimensions move with correct sign and magnitude.

● Pipeline is modular

Each stage swappable — easy to substitute SABR-LMM, HW2F, etc.

Questions?